Chaotic diffusion in the outer solar system

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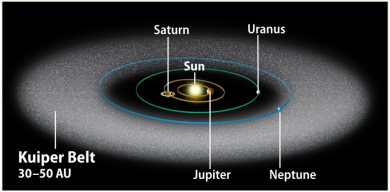
Trans-Neptunain region



Credit: astronomy.com

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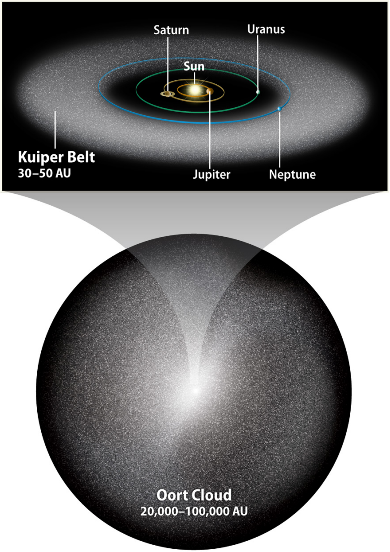
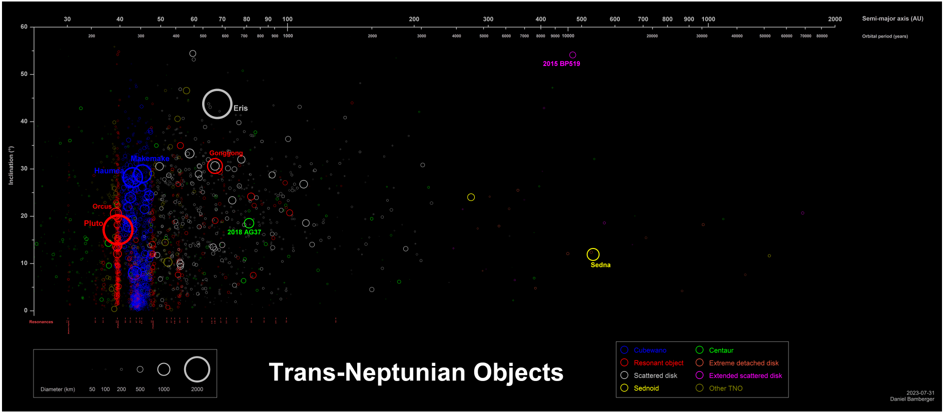


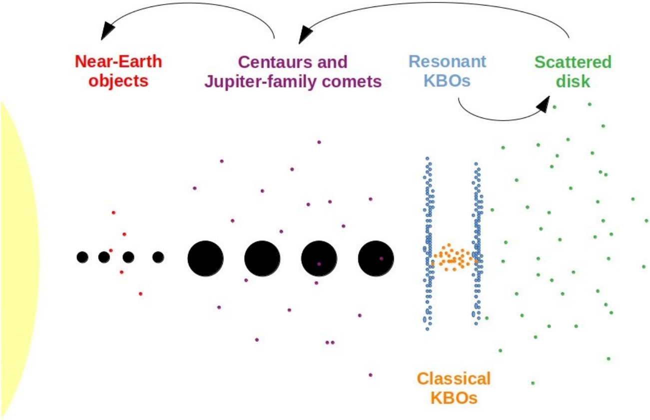
Credit: astronomy.com

Trans-Neptunian objects (TNOs)



Credit: Wikipedia

Role of chaotic diffusion in the transportation of TNOs

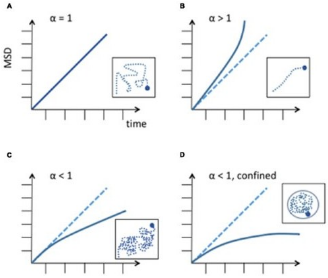


Chaotic diffusion

- drift of a phase point in the phase space "due to" nonlinearities in the equations of motion
- mean squared displacement (MSD) of an ensemble of particles ($x \equiv (x_i)_{i=1}^{N}$) in time:

$$\mathrm{MSD}_x(t) \equiv \left\langle (x(t) - x(0))^2 \right\rangle = 2dD_\alpha t^\alpha,$$

  where: d: dimension of x,
  α: diffusion exponent,
  D_α: diffusion coefficient
- nature of diffusion:
  normal ($\alpha = 1$)
  anomalous ($\alpha > 1$: super-, $\alpha < 1$: sub-)



Credit: Spichal & Fabre (2017)

MSD-based measure of diffusion

- fit of the linear function $\log(t) \mapsto \log(\mathrm{MSD}_x(t))$
- slope of fit: α, from intercept: D_α
- define a characteristic time of diffusion:

$$\tau = \left(\frac{\Delta^2}{2dD_\alpha}\right)^{1/\alpha},$$

  where: Δ^2: characteristic MSD of free choice

- choice of free parameters:

  $x = (L, G, H)$, $d = 3$, $L = \sqrt{\mu a}$ (total mechanical energy of orbit),

  $G = L\sqrt{1-e^2}$ (magnitude of angular momentum),

  $H = G\cos(I)$ (vertical component of ang. momentum)

  Δ: from Hill's stability criterium: $\Delta_a = 2\sqrt{3}R_{\mathrm{H}}$ (R_{H}: Hill radius of Neptune),

$$\Delta_L = \frac{\Delta_a}{2}\frac{\mu}{a},$$

$$\Delta_G = \Delta_L\sqrt{1-e^2},$$

$$\Delta_H = \Delta_G\cos(I),$$

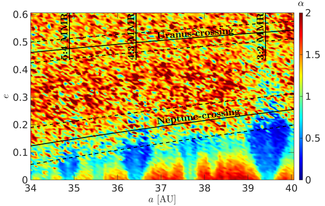
$$\Delta^2 = \Delta_L^2 + \Delta_G^2 + \Delta_H^2$$

Computations

Structure of dynamical maps:

- 100×100 "big" cells in the $a = [34, 40]$ AU, $e = [0, 0.6]$ plane
- each big cell is divided into 5×4 (in a and e) "small" cells
- altogether: 10 000 ensembles, with 20 test particles in each
- angular variables of one ensemble:

$$I = [0^\circ, 30^\circ],$$
$$\omega = [0^\circ, 360^\circ],$$
$$M = [0^\circ, 360^\circ],$$
$$\Omega = 0^\circ$$

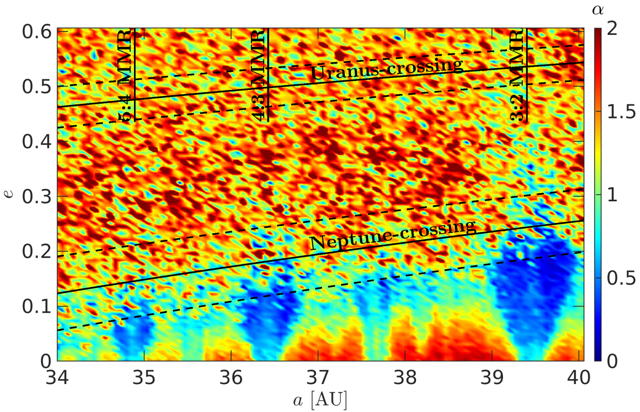


Numerical integrations:

- system: Sun + 4 giants + Pluto + test particles
- total integration time: $2 \cdot 10^5$ years
- sampling time-step: 100 years

Diffusion exponents

$$\mathrm{MSD}_x(t) = 2dD_\alpha t^{\alpha}$$



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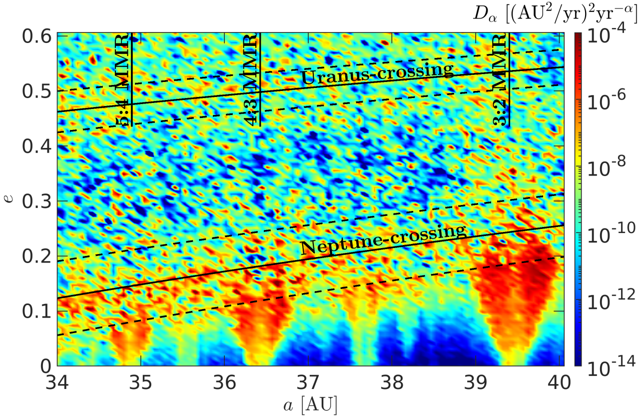
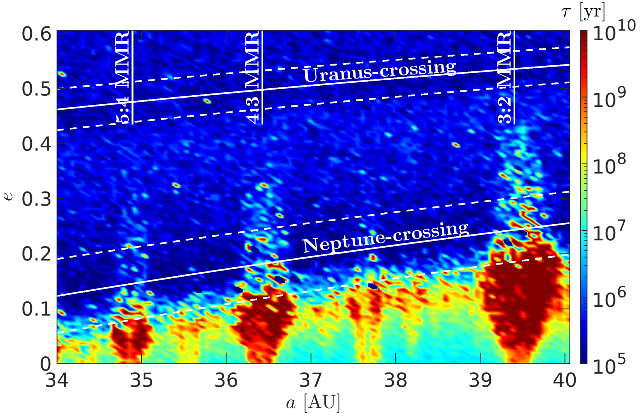
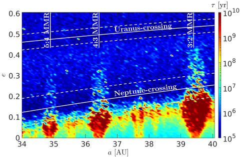


Diffusion timescales

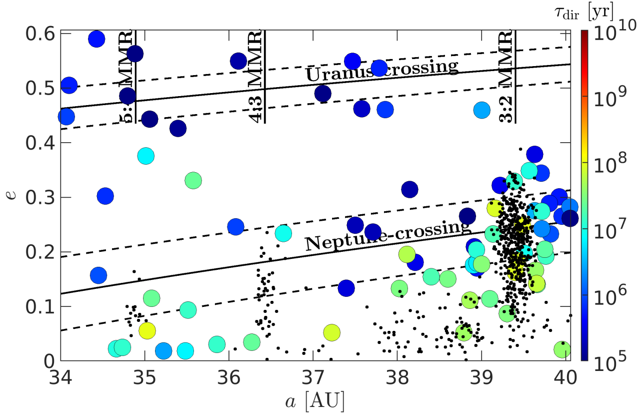
$$\tau = \left(\frac{\Delta^2}{2dD_\alpha}\right)^{1/\alpha}$$



Comparison with direct results (10^8-yr-long integration of real TNOs)



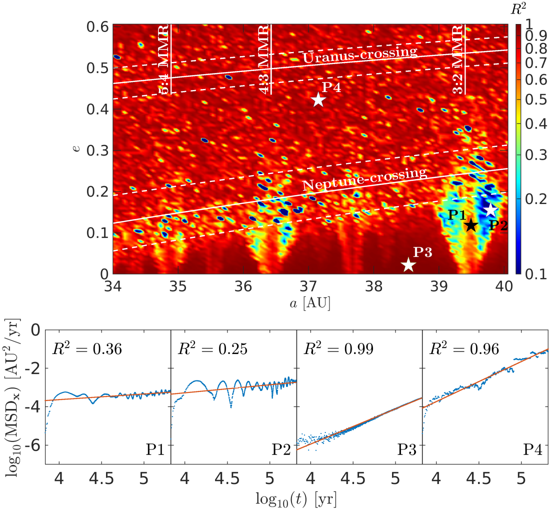
$\tau_{\rm dir}$: the time during which a given TNO moves away from its initial position (L, G, H) by Δ



Goodness of fit

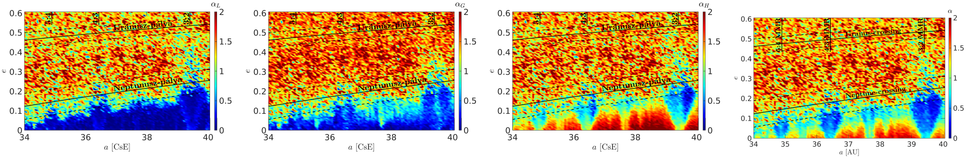
SS_{res}: sum of the squared differences between data points and fitted values
SS_{tot}: sum of the squared differences between data points and their mean value

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

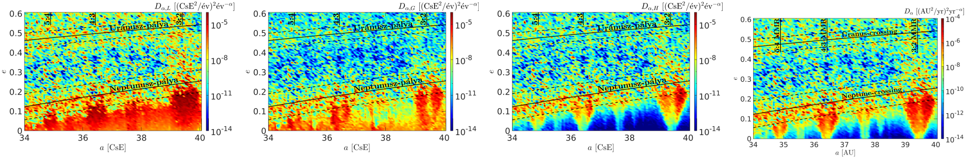


Anisotropy of diffusion?

Diffusion exponents α separately in L, G, and H (and in (L, G, H)):



Diffusion coefficients D_α separately in L, G, and H (and in (L, G, H)):



Summary

Conclusions:

- MSD method: simple way of quantifying the properties of chaotic diffusion
- diffusion-wise characteristics of the $34 - 40$ AU region:
  - (first-order) mean-motion resonances: sub-diffusion with long diffusion times
  - non-resonant large-eccentricity regions: less stable zones with super-diffusion
  - non-resonant circular orbits: complex dynamics, importance of (initial) inclinations

    $\rightsquigarrow$ slight discrepancy between indirect and direct results

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Thank you for your attention!